Sample Problem Sheet

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1. Given
$$\begin{aligned}\lim_{x\to 0}\frac{\cos x - 1}{x} &= 0\\ \lim_{x\to 0}\frac{\sin x}{x} &= 1\end{aligned}$$
differentiate from first principles $f(x) = \cos x$.

2. Differentiate the following functions:
   (a) $y = \arcsin(x)$
   (b) $f(x) = g(x)\ln(g(x))$.
   (c) $y = \exp(4x)$
   (d) $y = 2x^3 + 6x - 1$
   (e) $y = \frac{\sin x}{x}$.

3. Find the gradient of the unit circle ($x^2 + y^2 = 1$).

4. Find $\frac{dy}{dx}$, given
$$y^2 = \frac{x^3}{2-x}$$

5. A coin is weighted so that heads is four times as likely as tails. Find the probability that: (a) tails appears, (b) heads appears

6. Under which of the following functions does $S = \{a_1, a_2\}$ become a probability space?
   (a) $P(a_1) = \frac{1}{3}$, $P(a_2) = \frac{1}{2}$ (b) $P(a_1) = \frac{3}{4}$, $P(a_2) = \frac{1}{4}$
   (c) $P(a_1) = 1$, $P(a_2) = 0$ (d) $P(a_1) = \frac{5}{4}$, $P(a_2) = -\frac{1}{4}$

7. Which of the following is the derivative of $x\sin(x)$? (Circle the correct answer.)
   A $\sin(x)$
   B $x\cos(x)$
   C $\sin(x) + x\cos(x)$

8. Describe what is meant by the term *inheritance* in object-oriented programming. Use examples.